JEE Advanced 2026

Sample Paper - 1 (Paper-1)

Time Allowed: 3 hours **Maximum Marks: 180**

General Instructions:

This question paper has THREE main sections and four sub-sections as below.

MRQ

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

MCQ

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

NUM

- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

MATCH

- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- You will get +4 marks for the correct response and -1 for the incorrect response.

Physics

1. According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photoelectrons from a metal versus frequency of the incident radiation gives a straight line whose slope: **[3]**

a) depends on both the intensity of radiation and the nature of metal

b) depends on the nature of metal used

used

c) is the same for all metals and independent of the intensity of radiation

d) depends on the intensity of radiation

2. In a p-n junction diode not connected to any circuit **[3]**

a) the potential is the same everywhere.

b) there is an electric field at the junction directed from the p-type side to the n-type side.

c) the p-type side is at a higher potential than the n-type side.

d) there is an electric field at the junction directed from the H-side to the p-type side.

3. From a height, 3 balls are thrown with speed u, one vertically upward, second horizontally, third downward with times of fall be t_1,t_2 and t_3 respectively, then: **[3]**

a) $t_2 = \sqrt{t_1 t_3}$

b) $t_2 = \sqrt{t_1 + t_3}$

c) $t_2 = \frac{t_1+t_3}{2}$

d) $t_2 = \frac{2t_1 t_3}{t_1+t_3}$

4. A coil of wire having finite inductance and resistance has a conducting ring placed co-axially within it. The coil is connected to a battery at time t = 0, so that a time dependent current $I_1(t)$ starts flowing through the coil. If $I_2(t)$ is the current induced in the ring and B (t) is the magnetic field at the axis of the coil due to $I_1(t)$, then as a function of time (t > 0), the product $I_2(t)$ B (t) **[3]**

a) increases with time

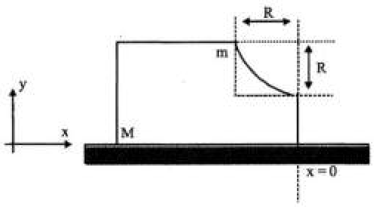
b) passes through a maximum

c) decreases with time

d) does not vary with time

5. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that **[4]**

instant, which of the following options is/are correct?



a) The position of the point mass m is: $x = -\sqrt{2}\frac{mR}{M+m}$

b) The velocity of the point mass m is: $v = \upsilon = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$

c) The velocity of the block M is: $V = -\frac{m}{M}\sqrt{2gR}$

d) The x component of displacement of the center of mass of the block M is: $-\frac{mR}{M+m}$

6. A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct? **[4]**

[ε_0 is the permittivity of free space]

a) If $h > 2R$ and $r > R$ then $\phi = \frac{Q}{\varepsilon_0}$

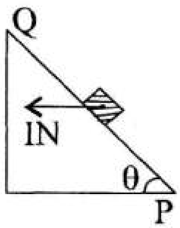
b) If $h < \frac{8R}{5}$ and $r = \frac{3R}{5}$ then $\phi = 0$

c) If $h > 2R$ and $r > \frac{4R}{5}$ then $\phi = \frac{Q}{5\varepsilon_0}$

d) If $h > 2R$ and $r = \frac{3R}{5}$ then $\phi = \frac{Q}{5\varepsilon_0}$

7. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. **[4]**

The block remains stationary if (take $g = 10\ m/s^2$)



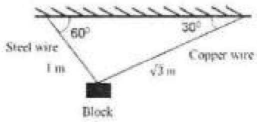
a) $\theta > 45°$ and a frictional force acts on the block towards P.

b) $\theta = 45°$

c) $\theta > 45°$ and a frictional force acts on the block towards Q.

d) $\theta < 45°$ and a frictional force acts on the block towards Q.

8. A block of weight 100N is suspended by copper and steel wires of same cross-sectional area 0.5 cm^2 and, length $\sqrt{3}$m and 1m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with the ceiling are 30° and 60°, respectively. If elongation in copper wire is and elongation in steel wire is (Δl_S), then the ratio $\frac{\Delta l_c}{\Delta l_s}$ is ________. **[4]**
[Young's modulus for copper and steel are 1×10^{11} N/m^2 and 2×10^{11} N/m^2, respectively.]

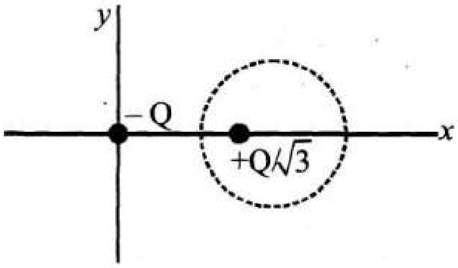


9. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice. **[4]**
[Take atmospheric pressure = 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension.]

10. A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} Nm rad^{-1}. When a current flows through the galvanometer, a full-scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 - 1.0 A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in ohms, is ________. **[4]**

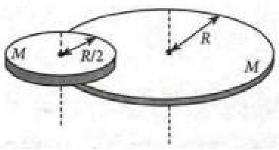
11. Two point charges -Q and $+\frac{Q}{\sqrt{3}}$ are placed in the xy-plane at the origin (0, 0) and a point (2, 0), respectively, as shown in the figure. This results in an equipotential circle **[4]**

of radius R and potential V = 0 in the xy-plane with its center at (b, 0). All lengths are measured in meters.



The value of b is ________ meter.

12. A disc of mass M and radius R is free to rotate about its vertical axis as shown in the figure. **[4]**



A battery operated motor of negligible mass is fixed to this disc at a point on its circumference. Another disc of the same mass M and radius R/2 is fixed to the motor's thin shaft. Initially, both the discs are at rest. The motor is switched on so that the smaller disc rotates at a uniform angular speed ω. If the angular speed at which the large disc rotates is ω/n, then the value of n is ________.

13. A series R-C combination is connected to an AC voltage of angular frequency ω = 500 radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is **[4]**

14. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists: **[4]**

List I	**List II**
P. Alpha decay	1. ${}^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{O} + \ldots$
Q. $\beta+$ decay	2. ${}^{138}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + \ldots$
R. Fission	3. ${}^{185}_{83}\mathrm{Bi} \rightarrow {}^{184}_{82}\mathrm{Pb} + \ldots$
S. Proton emission	4. ${}^{239}_{94}\mathrm{Pu} \rightarrow {}^{140}_{57}\mathrm{La} + \ldots$

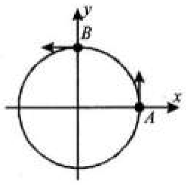
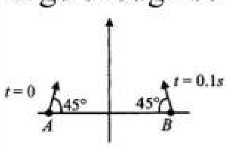
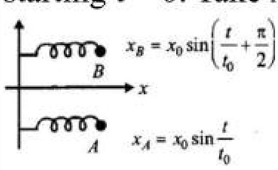
a) (P) - (2); (Q) - (1); (R) - (4); (S) - (3)

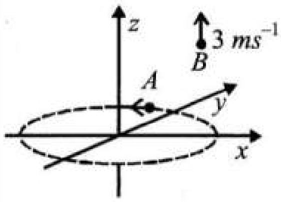
b) (P) - (1); (Q) - (3); (R) - (2); (S) - (4)

c) (P) - (4); (Q) - (3); (R) - (2); (S) - (1)

d) (P) - (4); (Q) - (2); (R) - (1); (S) - (3)

15. List I describes four systems, each with two particles A and B in relative motion as shown in figures. List II gives possible magnitudes of their relative velocities (in m s^{-1}) at time t = $\frac{\pi}{3}$ S. **[4]**

List-I	List-II
(I) A and B are moving on a horizontal circle of radius 1 m with uniform angular speed ω = 1 rad s^{-1}. The initial angular positions of A and B at time t = 0 are $\theta = 0$ and $\theta = \frac{\pi}{2}$, respectively. 	(P) $\frac{\sqrt{3}+1}{2}$
(II) Projectiles A and B are fired (in the same vertical plane) at t = 0 and t = 0.1 s respectively, with the same speed v = $\frac{5\pi}{\sqrt{2}}$ m s^{-1} and at 45^{o} from the horizontal plane. The initial separation between A and B is large enough so that they do not collide. (g = 10 m s^{-2}). 	(Q) $\frac{(\sqrt{3}-1)}{\sqrt{2}}$
(III) Two harmonic oscillators A and B moving in the x direction according to $x_A = x_0 \sin \frac{t}{t_0}$ and $x_B = x_0 \sin\left(\frac{t}{t_0} + \frac{\pi}{2}\right)$ respectively, starting t = 0. Take x_0 = 1 m, t_0 = 1 s. 	(R) $\sqrt{10}$
(IV) Particle A is rotating in a horizontal circular path of radius 1 m on the xy plane, with constant angular speed ω = 1 rad s^{-1}. Particle B is moving up at a constant speed 3 ms^{-1} in the vertical direction as shown	(S) $\sqrt{2}$

in the figure. (Ignore gravity.) 	
	(T) $\sqrt{25\pi^2 + 1}$

Which one of the following options is correct?

a) (I) → (T); (II) → (P); (III) → (R); (IV) → (S)

b) (I) → (R); (II) → (T); (III) → (P); (IV) → (S)

c) (I) → (S); (II) → (P); (III) → (Q); (IV) → (R)

d) (I) → (S); (II) → (T); (III) → (P); (IV) → (R)

16. A person in lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List-I and the distance where the water jet hits the floor of the lift is given in List-II. **[4]**

Match the statements from List-I with those in List-II and select the correct answer using the code given below the lists.

List - I	**List - II**
(P) Lift is accelerating vertically up	(1) d = 1.2 m
(Q) Lift is accelerating vertically down with an acceleration less than the gravitational acceleration	(2) d > 1.2 m
(R) Lift is moving vertically up with constant speed	(3) d < 1.2 m
(S) Lift is falling freely	(4) No water leaks out of the jar

a) P - 1, Q - 1, R - 1, S - 4

b) P - 2, Q - 3, R - 1, S - 4

c) P - 2, Q - 3, R - 2, S - 4

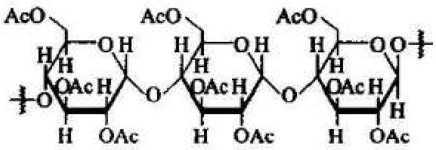
d) P - 2, Q - 3, R - 1, S - 1

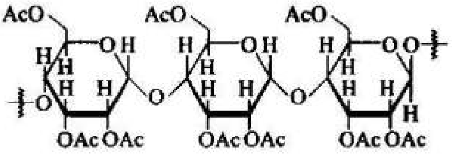
Chemistry

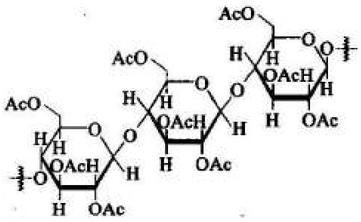
17. $\Delta H_{vap} = 30$ kJ/mol and $\Delta S_{vap} = 75$ Jmol^{-1} K^{-1}. Find the temperature of vapour, at one atmosphere [3]

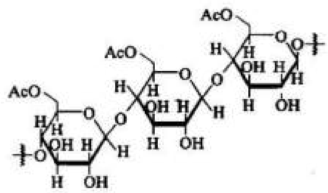
a) 250 K
b) 400 K
c) 298 K
d) 350 K

18. Cellulose upon acetylation with excess acetic anhydride/ H_2SO_4 (catalytic) gives cellulose triacetate whose structure is [3]

a) 

b) 

c) 

d) 

19. Which one is solder? [3]

a) Fe & Zn
b) Pb & Sn
c) Cu & Pb
d) Zn & Cu

20. In a metal deficient oxide sample, $M_X Y_2 O_4$ (M and Y are metals), M is present in both +2 and +3 oxidation states and Y is in +3 oxidation state. If the fraction of M^{2+} ions present in M is $\frac{1}{3}$, the value of X is ________. [3]

a) 0.25
b) 0.67
c) 0.75
d) 0.33

21. According to the Arrhenius equation, [4]

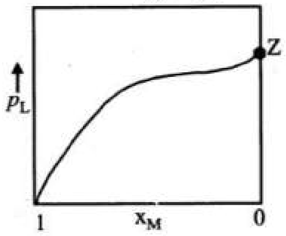
a) a high activation energy usually implies a fast reaction.
b) rate constant increases with increase in temperature. This is due to a greater number of

collisions whose energy exceeds the activation energy.

c) the pre-exponential factor is a measure of the rate at which collisions occur, irrespective of their energy.

d) higher the magnitude of activation energy, stronger is the temperature dependence of the rate constant

22. For a solution formed by mixing liquids L and M, the vapour pressure of L plotted against the mole fraction of M in solution is shown in the following figure. Here x_L and x_M represent mole fractions of L and M, respectively, in the solution. The correct statement(s) applicable to this system is (are) **[4]**



a) The point Z represents vapour pressure of pure liquid L and Raoult's law is obeyed when $x_L \to 1$.

b) The point Z represents vapour pressure of pure liquid M and Raoult's law is obeyed when $x_L \to 0$

c) Attractive intermolecular interactions between L-L in pure liquid L and M-M in pure liquid M are stronger than those between L-M when mixed in solution.

d) The point Z represents vapour pressure of pure liquid M and Raoult's law is obeyed from $x_L = 0$ to $x_L = 1$.

23. Sodium nitrate decomposes above 800^{o} C to give **[4]**

a) Na_2O

b) O_2

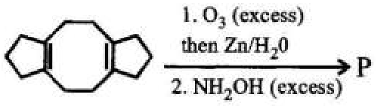
c) NO_2

d) N_2

24. Consider the following molecules: Br_3O_8, F_2O, $H_2S_4O_6$, $H_2S_5O_6$, and C_3O_2. Count the number of atoms existing in their zero oxidation state in each molecule. **[4]**

Their sum is ________.

25. In the reaction given below, the total number of atoms having sp^2 hybridization in the major product P is ________ **[4]**



26. 20 mL of 0.02 M $K_2Cr_2O_7$ solution is used for the titration of 10 mL of Fe^{2+} solution in the acidic medium. The molarity of Fe^{2+} solution is ________ $\times 10^{-2}$ M. (Nearest Integer) **[4]**

27. $CH_3-CH=CH_2 \xrightarrow[CCl_4]{Br_2} A \xrightarrow{NaNH_2} B \xrightarrow{CH_3-Cl} C$ **[4]**
C gives Br_2 - water test, but does not give test with ammonical silver nitrate, calculate the mass of $NaNH_2$ needed to form 1 mole of 'C'.

28. To a 25mL H_2O_2 solution, excess of acidified solution of potassium iodide was added. The iodine liberated required 20 mL of 0.3 N sodium thiosulphate solution. Calculate the volume strength of H_2O_2 solution. **[4]**

29. Total number of geometrical isomers for the complex $[RhCl(CO)(PPh_3)(NH_3)]$ is **[4]**

30. Dilution processes of different aqueous solutions, with water, are given in LIST-I. The effects of dilution of the solutions on $[H^+]$ are given in LIST-II. **[4]**
(Note: Degree of dissociation (α) of weak acid and weak base is << 1; degree of hydrolysis of salt << 1; $[H^+]$ represents the concentration of H^+ ions)

LIST-I	LIST-II
(P) (10 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 60 mL	(1) the value of $[H^+]$does not change on dilution
(Q) (20 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 80 mL	(2) the value of $[H^+]$ changes to half of its initial value on dilution
(R) (20 mL of 0.1 M HCl + 20 mL of 0.1 M ammonia solution) diluted to 80 mL	(3) the value of $[H^+]$ changes to two times of its initial value on dilution
(S) 10 mL saturated solution of $Ni(OH)_2$ in equilibrium with excess solid $Ni(OH)_2$ is diluted to 20	(4) the value of $[H^+]$ changes to $\frac{1}{\sqrt{2}}$ times of its initial

mL (solid $Ni(OH)_2$ is still present after dilution).	value on dilution
	(5) the value of $[H^+]$ changes to $\sqrt{2}$ times of its initial value on dilution

a) P- 4; Q - 2; R - 3; S - 1

b) P - 1; Q - 4; R - 5; S - 3

c) P - 4; Q - 3; R - 2; S - 3

d) P - 1; Q - 5; R - 4; S - 1

31. Match List-I with List-II **[4]**

List-I **Vitamin**	**List-II** **Deficiency disease**
(A) Vitamin A	(I) Beri-Beri
(B) Thiamine	(II) Cheilosis
(C) Ascorbic acid	(III) Xerophthalmia
(D) Riboflavin	(IV) Scurvy

a) A - IV, B - II, C - III, D - I

b) A - III, B - II, C - IV, D - I

c) A - IV, B - I, C - III, D - II

d) A - III, B - I, C - IV, D - II

32. An aqueous solution of X is added slowly to an aqueous solution of Y as shown in List I. The variation in conductivity of these reactions is given in List II. Match list I with List II and select the correct answer using the code given below the lists: **[4]**

List-I	**List-II**
(P) $\underset{X}{(C_2H_5)_3N} + \underset{Y}{CH_3COOH}$	(1) Conductivity decreases and then increases
(Q) $\underset{X}{KI(0.1M)} + \underset{Y}{AgNO_3(0.01M)}$	(2) Conductivity decreases and then does not change much
(R) $\underset{X}{CH_3COOH} + \underset{Y}{KOH}$	(3) Conductivity increases and then does not change much
(S) $\underset{X}{NaOH} + \underset{Y}{HI}$	(4) Conductivity does not change much and then increases

a) (P) - (4), (Q) - (3), (R) - (2), (S) - (1)

b) (P) - (2), (Q) - (3), (R) - (4), (S) - (1)

c) (P) - (3), (Q) - (4), (R) - (2), (S) - (1)

d) (P) - (1), (Q) - (4), (R) - (3), (S) - (2)

Maths

33. The domain of definition of f (x) = $\frac{\log_2(x+3)}{x^2+3x+2}$ is **[3]**

a) $(-2, \infty)$

b) $\frac{R}{(-1,-2,-3)}$

c) $\frac{R}{(-1,-2)}$

d) (- 3, ∞) - {-1, -2}

34. Let a tangent to the curve $y^2 = 24x$ meet the curve xy = 2 at the points A and B. Then the mid points of such line segments AB lie on a parabola with the **[3]**

a) directrix 4x = ·3

b) Length of latus rectum 2

c) directrix 4x = 3

d) Length of latus rectum $\frac{3}{2}$

35. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$ **[3]**

a) f(x) has a local maxima

b) f(x) is strictly decreasing function

c) f(x) is bounded

d) f(x) is strictly increasing function

36. The expression $\left[x + \left(x^3 - 1\right)^{\frac{1}{2}}\right]^5 + \left[x - \left(x^3 - 1\right)^{\frac{1}{2}}\right]^5$ is a polynomial of degree **[3]**

a) 6

b) 8

c) 7

d) 5

37. Let f:[0, 1] → [0, 1] be the function defined by $f(x) = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$. Consider the square region S = [0 ,1] × [0, 1]. Let G = {(x, y) ∈ S : y > f(x)} be called the green region and R = {(x, y) ∈ S : y < f(x)}, be called the red region. Let L_h = {(x, h) ∈ [0, 1]}, be the horizontal line drawn at a height h ∈ [0. 1]. Then which of the following statements is (are) true? **[4]**

a. There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the green region below the line L_h

b. There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the red region below the line L_h

c. There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the red region below the line L_h

d. There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the green region below the line L_h

a) Statement (b) is true. b) Statement (c) is true.

c) Statement (d) is true. d) Statement (a) is true.

38. In R^3, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be the plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2 , then which of the following relations is (are) true? **[4]**

a) $2\alpha - \beta + 2\gamma + 4 = 0$ b) $2\alpha - \beta + 2\gamma - 8 = 0$

c) $2\alpha + \beta - 2\gamma - 10 = 0$ d) $2\alpha + \beta + 2\gamma + 2 = 0$

39. Let $S_1 = \{(i, j, k): i, j, k \in \{1, 2,..., 10\}\}$ **[4]**

$S_2 = \{(i, j): 1 \leq i < j + 2 \leq 10, i, j \in \{1, 2,..., 10\}\}$,

$S_3 = \{(i, j, k, l): 1 \leq i < j < k < l, i, j, k, l \in \{1, 2,..., 10\}\}$.

and $S_4 = \{(i, j, k, l)$: i, j, k and l are distinct elements in $\{1, 2,..., 10\}\}$.

If the total number of elements in the set S_r is n_r, r = 1, 2, 3, 4, then which of the following statements (are) **TRUE?**

a) $n_2 = 44$ b) $n_1 = 1000$

c) $n_3 = 220$ d) $\frac{n_4}{12} = 420$

40. Let $f : R \to R$ be a continous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^{x} f(t)dt$ for all $x \in [-1, 2]$ and $G(x) = \int_{-1}^{x} t|f(f(t))|dt$ for all x [-1, 2]. If $\lim_{x \to 1} \frac{F(x)}{G(x)} = \frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is: **[4]**

41. Let X be a random variable, and let $P(X = x)$ denote the probability that X takes the value x. Suppose that the points $(x, P(X = x)), x = 0, 1, 2, 3, 4$, lie on a fixed straight line in the xy-plane, and $P(X = x) = 0$ for all $x \in \mathbb{R} - \{0, 1, 2, 3, 4\}$. If the mean of X is $\frac{5}{2}$, and the variance of X is α, then the value of 24α is ________. **[4]**

42. Let z = $\frac{-1+\sqrt{3}i}{2}$, where i = $\sqrt{-1}$, and r, s $\in \{1, 2, 3\}$. Let P = $\begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which P^2 = -I is **[4]**

43. The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side? **[4]**

44. For x $\in \mathbb{R}$, then number of real roots of the equation $3x^2 - 4\left|x^2 - 1\right| + x - 1 = 0$ is ________. **[4]**

45. Let $f(x) = x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that $f(1) = -9$. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3 + 3ax^2 + 2bx = 0$, where $i = \sqrt{-1}$. If $\alpha_1, \alpha_2, \alpha_3$, and α_4 are all the roots of the equation $f(x) = 0$, then $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$ is equal to ________. **[4]**

46. Let $f_1 : R \to R, f_2 : [0, \infty) \to R, f_3 : R \to R$ and $f_4 : R \to [0, \infty)$ be defined by **[4]**

$f_1(x) = \begin{cases} |x|, \text{ if } x < 0, \\ e^x, \text{ if } x \geq 0; \end{cases}$ $f_2(x) = x^2; f_3(x) = \begin{cases} \sin x, \text{ if } x < 0 \\ x, \text{ if } x \geq 0 \end{cases}$ and

$f_4(x) = \begin{cases} f_2(f_1(x)), & \text{if } x < 0, \\ f_2(f_1(x)) - 1, & \text{if } x \geq 0. \end{cases}$

LIST - I	**LIST - II**
(P) f_4 is	(1) Onto but not one-one
(Q) f_3 is	(2) Neither continuous nor one-one
(R) $f_2 o f_1$ is	(3) Differentiable but not one-one
(S) f_2 is	(4) Continuous and one-one

a) P → 3; Q → 1; R → 4; S → 2 b) P → 1; Q → 3; R → 4; S → 2

c) P → 1; Q → 3; R → 2; S → 4 d) P → 3; Q → 1; R → 2; S → 4

47. Consider the given data with frequency distribution **[4]**

x_i	3	8	11	10	5	4
f_i	5	2	3	2	4	4

Match each entry in **List-I** to the correct entries in **List-II**.

List-I	**List-II**
(P) The mean of the above data is	(1) 2.5
(Q) The median of the above data is	(2) 5
(R) The mean deviation about the mean of the above data is	(3) 6
(S) The mean deviation about the median of the above data is	(4) 2.7
	(5) 2.4

a) (P) → (3), (Q) → (2), (R) → (4), (S) → (5)

b) (P) → (3), (Q) → (2), (R) → (1), (S) → (5)

c) (P) → (2), (Q) → (3), (R) → (4), (S) → (1)

d) (P) → (3), (Q) → (3), (R) → (5), (S) → (5)

48. Let z be a complex number satisfying $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$, where $\bar{z}$ denotes the complex conjugate of z. Let the imaginary part of z be non-zero. Match each entry in **List-I** to the correct entries in **List-II.** [4]

List-I	**List-II**
(P) $\|z\|^2$ is equal to	(1) 12
(Q) $\|z-\bar{z}\|^2$ is equal to	(2) 4
(R) $\|\bar{z}\|^2 + \|z+\bar{z}\|^2$ is equal to	(3) 8
(S) $\|z+1\|^2$ is equal to	(4) 10
	(5) 7

a) (P) → (2), (Q) → (3), (R) → (5), (S) → (4)

b) (P) → (2), (Q) → (1), (R) → (3), (S) → (5)

c) (P) → (1), (Q) → (3), (R) → (5), (S) → (4)

d) (P) → (2), (Q) → (4), (R) → (5), (S) → (1)

Solution

Physics

1.

(c) is the same for all metals and independent of the intensity of radiation

Explanation:

According to Einstein's photoelectric equation

$(KE)_{max.}$ = hv - W

The slope of the line in the graph is h, the Planck's constant.

2.

(d) there is an electric field at the junction directed from the H-side to the p-type side.

Explanation:

At junction a potential barrier/depletion layer is formed, with n-side at higher potential and p-side at lower potential. Therefore, there is an electric field at the junction directed from the n-side to p-side.

3. **(a)** $t_2 = \sqrt{t_1 t_3}$

Explanation:

$-s = ut_1 - \frac{1}{2}gt_1^2$ ...(i)

$-s = ut_3 - \frac{1}{2}gt_3^2$ ...(ii)

$-s = -\frac{1}{2}gt_2^2$ ...(iii)

$-st_3 = ut_1t_3 - \frac{1}{2}gt_1^2t_3$ ...(iv)

$-st_1 = -ut_1t_3 - \frac{1}{2}gt_3^2t_1$ (v)

Adding, eqns. (iv) and (v),

-s $(t_1 + t_3) = -\frac{1}{2}$ gt_3t_1 $(t_3 + t_1)$

or s = $+\frac{1}{2}$ gt_3t_1 ...(vi)

From eqns. (iii) and (vi),

$\frac{1}{2}gt_3t_1 = \frac{1}{2}gt_2^2$

$\therefore t_2 = \sqrt{t_3t_1}$

4.

(b) passes through a maximum

Explanation:

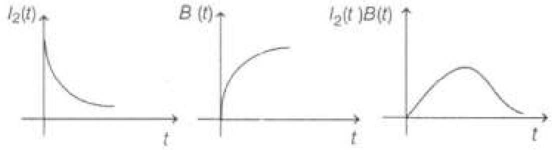
The equation of I_1(t), I_2(t) and B(t) will take the following forms:

$I_1(t) = K_1\left(1 - e^{-k_2 t}\right) \rightarrow$ current growth in L-R circuit

$B(t) = K_3\left(1 - e^{-k_2 t}\right) \rightarrow B(t) \propto I_1(t)$

$I_2(t) = K_4 e^{-k_2 t}$ $[I_2(t) = \frac{e_2}{R}$ and $e_2 \propto \frac{dI_1}{dt} : e_2 = -M\frac{dI_1}{dt}]$

Therefore, the product $I_2(t)B(t) = K_5 e^{-k_2 t}\left(1 - e^{-k_2 t}\right)$. The value of this product is zero at t = 0 and t = ∞. Therefore, the product will pass through a maximum value. The corresponding graphs will be as follows:



5. **(b)** The velocity of the point mass m is: v = $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$

(d) The x component of displacement of the center of mass of the block M is: $-\frac{mR}{M+m}$

Explanation: Let the block of mass M moves by distance x towards left.

$M_X = m(R - x)$

$\Rightarrow$ x = $\frac{mR}{M+m}$ towards left $\therefore$ x = $-\frac{mR}{M+m}$

If v is the velocity of mass 'm' as it leaves the block and V is the velocity of block at that instant then according to conservation of linear momentum

mv = MV

By energy conservation

mgR = $\frac{1}{2}mv^2 + \frac{1}{2}MV^2$

Solving we get, v = $\sqrt{\frac{2gR}{1+\frac{m}{M}}}$ and V = $\frac{m}{M}\sqrt{\frac{2gR}{1+\frac{m}{M}}}$

6. **(a)** If h > 2R and r > R then $\phi = \frac{Q}{\varepsilon_0}$

(b) If h < $\frac{8R}{5}$ and r = $\frac{3R}{5}$ then $\phi = 0$

(d) If h > 2R and r = $\frac{3R}{5}$ then $\phi = \frac{Q}{5\varepsilon_0}$

Explanation:

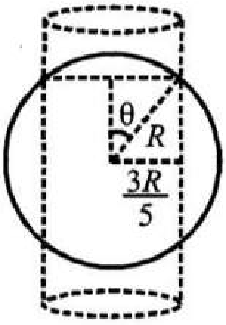
a. for h > 2R and r = $\frac{3R}{5}$

$\sin\theta = \frac{\frac{3R}{5}}{R} = \frac{3}{5} = 37^{\circ}$

q_{in} = Q[1 − cos 37°] = Q$\left[1 - \frac{4}{5}\right] = \frac{Q}{5}$

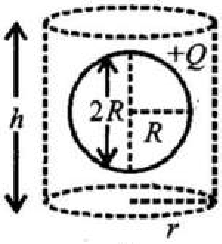
From Gauss's theorem Q = $\frac{q_{in}}{\varepsilon_0}$

$\therefore \phi = \frac{Q}{5\epsilon_0}$



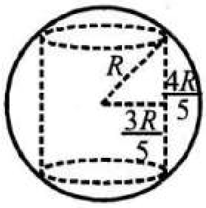
b. for h > 2R and r > R

$\phi = \frac{q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0}$



c. For h < $\frac{8}{5}$R and r = $\frac{3}{5}$R

$\phi = \frac{q_{in}}{\epsilon_0} = 0$

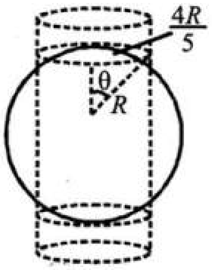


d. For h > 2R and r > $\frac{4}{5}$R

$\sin\theta = \frac{\frac{4R}{5}}{R} = \frac{4}{5} = 0.8 \Rightarrow \theta = 53^{\circ}$

$q_{in} = Q[1 - \cos\theta] = Q[1 - \frac{3}{5}] = \frac{2Q}{5}$

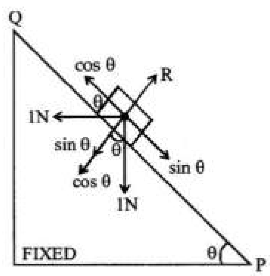
$\therefore \phi = \frac{2Q}{5\epsilon_0}$



7. **(b)** $\theta = 45°$

(c) $\theta > 45°$ and a frictional force acts on the block towards Q.

Explanation: The various forces acting on the block are as shown in the figure.



When $\theta = 45°$, $\sin\theta = \cos\theta$

The block will remain stationary and the frictional force is zero.

When $\theta > 45°$, $\sin\theta > \cos\theta$

Therefore a frictional force acts towards Q.

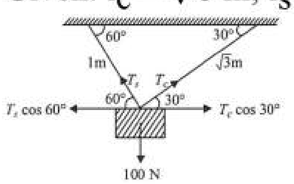
When $0 < 45°$, $\cos\theta > \sin\theta$

Therefore a frictional force acts towards P.

8. 2

Explanation:

Given: $l_c = \sqrt{3}$ m; $l_s = 1$ m; $Y_c = 1 \times 10^{11}$ N/m^2 and $T = 2 \times 10^{11}$ N/m^2.



At equilibrium, $T_S \cos 60° = T_C \cos 30°$

$\Rightarrow \frac{T_S}{2} = \frac{T_c\sqrt{3}}{2} \Rightarrow T_3 = \sqrt{3}T_r \Rightarrow \frac{T_c}{T_s} = \frac{1}{\sqrt{3}}$

$\therefore \frac{l_c}{l_s} = \frac{\sqrt{3}}{1}$ and $\frac{Y_c}{Y_s} = \frac{1\times10^{11}}{2\times10^{11}} = \frac{1}{2}$

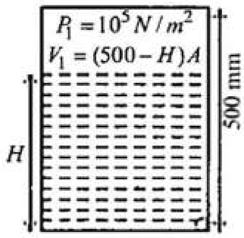
From, $Y = \frac{Fl}{A\Delta l} \Rightarrow \Delta l = \frac{Fl}{AY}$

Here, $A_S = A_C$

$\therefore \frac{\Delta l_c}{\Delta l_s} = \left(\frac{T_c}{T_s}\right) \times \left(\frac{l_c}{l_s}\right) \times \left(\frac{Y_s}{Y_c}\right) = \left(\frac{1}{\sqrt{3}}\right) \times \left(\frac{\sqrt{3}}{1}\right) \times \left(\frac{2}{1}\right) = 2$

9. 6

Explanation:



Initially

Initially, pressure of air column above water $P_1 = 10^5$ Nm^{-2} and volume $V_1 = (500 - H)A$, where A is the area of cross-section of the vessel.

Finally, the volume of air column above water $P_2 = (500 - 200)$ A = 300 A. If P_2 is the pressure of air then

$P_2 + \rho gh = P_1$

$\therefore P_2 + 10^3 \times 10 \times \frac{200}{1000} = 10^5$

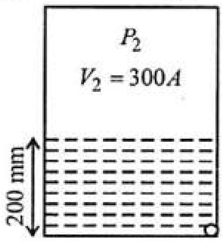
$\therefore P_2 = 9.8 \times 10^4 \text{ N/m}^2$

Assuming the temperature remains constant, according to Boyle's law

$P_1V_1 = P_2V_2$

$\therefore 10^5 \times (500 - H)A = (9.8 \times 10^4) \times 300 \text{ A} \Rightarrow H = 206 \text{ mm}$

$\therefore$ Fall in height of water level due to the opening of orifice 206 - 200 = 6 mm



Finally

10. 5.56

Explanation:

Given: B = 0.02T, C = 10^{-4} Nm

rad^{-1}

$\theta = 0.2$ rad

N = 50 and

$A = 2 \times 10^{-4} \text{ m}^2$

We know, $C\theta = NBA\, I_g$

$\therefore I_g = \frac{C\theta}{NBA} = \frac{10^{-4} \times 0.2}{50 \times 2 \times 10^{-4} \times 0.02} = 0.1 \text{ A}$

To convert a galvanometer to ammeter, a shunts is used in parallel to the galvanometer.

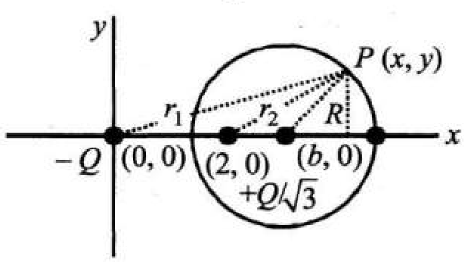
$I_g \times G = (I - I_g)S$

$\therefore S = \frac{I_g G}{I - I_g} = \frac{0.1 \times 50}{1 - 0.1} = \frac{50}{9} = 5.56\Omega$

11. 3.0

Explanation:

let us consider a point P on the circle



$V_P = 0 = \frac{k(-Q)}{r_1} + \frac{\frac{kQ}{\sqrt{3}}}{r_2} \Rightarrow \frac{kQ}{r_1} = \frac{\frac{kQ}{\sqrt{3}}}{r_2}$

$\Rightarrow \frac{1}{\sqrt{x^2+y^2}} - \frac{1}{\sqrt{3}\sqrt{(x-2)^2+y^2}}$

$\Rightarrow 3(x-2)^2 + 3y^2 = x^2 + y^2$

$\Rightarrow 3\left(x^2+4-4x\right) - x^2 + 2y^2 = 0 \Rightarrow 2x^2 + 12 - 12x + 2y^2 = 0$

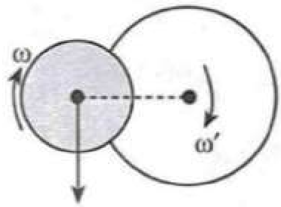
$\Rightarrow x^2 + 6 - 6x + y^2 = 0 \Rightarrow (x-3)^2 + y^2 = (\sqrt{3})^2$

or $(x-b)^2 + y^2 = (\sqrt{3})^2 = R^2$

$\therefore$ R = $\sqrt{3}$ = 1.73 and b = 3

12. 12

Explanation:



Let the angular velocity of the large disc be ω'.

Using conservation of angular momentum about the axis of bigger disc

$\frac{MR^2}{2}\omega' + MR^2\omega' - \frac{M(R/2)^2}{2}\omega = 0$

$\Rightarrow \frac{3}{2}MR^2\omega' = \frac{MR^2\omega}{8}$

$\omega' = \frac{\omega}{12}$, so, $n = 12$

13. 4

Explanation:

Time constant, T = RC

Impendance Z = $\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

Given Z = $R\sqrt{1.25}$

$\therefore R\sqrt{1.25} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

$\therefore$ RC = $\frac{2}{\omega} = \frac{2}{500} \times 1000$ ms $\therefore$ RC = 4 ms

14. **(a)** (P) - (2); (Q) - (1); (R) - (4); (S) - (3)

Explanation:

In B^+- decay mass number (Z) decreases by 1 and mass number (A) remains unchanged.

${}^{15}_{8}\mathrm{O} \longrightarrow {}^{15}_{7}\mathrm{N} + {}^{0}_{1}\beta$

In α-decay mass number (A) decreases by 4 unit and atomic number (Z) by 2 unit.

${}^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + \underset{2-\text{ particle}}{4\quad \mathrm{He}}$

In proton $\left({}_{1}^{1}\mathrm{H}\right)$ emission both (A) and (Z) decreases by 1 .

${}_{83}^{185}\mathrm{Bi} \longrightarrow {}_{82}^{184}\mathrm{Pb} + {}_{1}^{1}\mathrm{H}$

In fission process heavier nucleus breaks into two fragments.

${}_{94}^{239}\mathrm{Pu} \longrightarrow {}_{57}^{140}\mathrm{La} + {}_{37}^{99}\mathrm{X}$

15.

(d) (I) → (S); (II) → (T); (III) → (P); (IV) → (R)

Explanation:

(I) $V_{BA}^{2} = V_{A}^{2} + V_{B}^{2}$ - $2V_BV_A$ cos

As = , = 90^{o} remains constant

Also, $V_A = V_B = 1$ m/s [V = R]

V_{BA} = m/s. So I S.

(II)

= + =

=

After t = 0.1 sec, both projectile came in air. So, there relative acceleration is zero. So, relative velocity should not change after it.

$V_{rel} = V_{rel}$ (t = 0.1 sec) = = . So II T

(III) $x = x_A - x_B$

$= x_0$ sin t - $x_0$sin [$t_0 = 1$]

= sin t - cos t =

=

V_{rel} = = =

= = . So, III P

(IV) and are always perpendicular

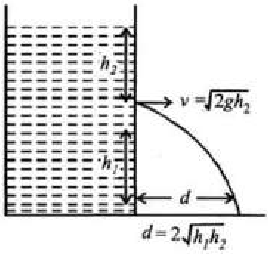
So, = = = m/s

So IV R

16. **(a)** P - 1, Q - 1,R - 1, S - 4

Explanation:

Horizontal distance,



d = y × t

$= \sqrt{2gh_2} \times \sqrt{\frac{2h_1}{g}}$

$= 2\sqrt{h_1 h_2}$

If $g_{eff} > g$

$g_{eff} = g$

$g_{eff} < g$

In all the three cases $d = 2\sqrt{h_1 h_2} = 1.2$ m

If $g_{eff} = 0$, then no water leaks out as there will be no pressure difference.

Chemistry

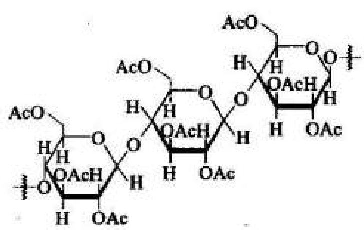
17.

(b) 400 K

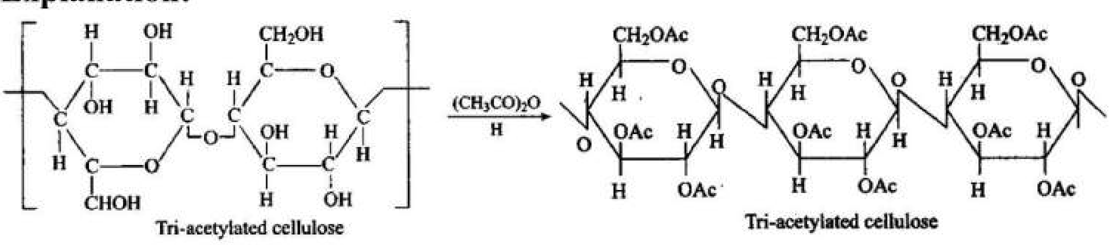
Explanation:

$T = \frac{\Delta H_{vap}}{\Delta S_{vap}} = \frac{30,000}{75} = 400$ K

18.

(c) 

Explanation:



19.

(b) Pb & Sn

Explanation:

Solder is an alloy containing Sn - 67% and Pb - 33%.

20.

(c) 0.75

Explanation:

In $M_X Y_2 O_4$,

Fraction of M^{2+} ions present in M is $\frac{X}{3}$.

Fraction of M^{3+} ions present in M is $\frac{2X}{3}$.

So, total oxidation states of all atoms,

$2 \times \frac{X}{3} + 3 \times \frac{2X}{3} + 2(+3) + 4(-2) = 0$
$\frac{2X}{3} + \frac{6X}{3} + 6 - 8 = 0; \frac{2X}{3} + \frac{6X}{3} = 2$
$\frac{8X}{3} = 2; 8X = 6; X = \frac{6}{8} = \frac{3}{4} = 0.75$

21. **(b)** rate constant increases with increase in temperature. This is due to a greater number of collisions whose energy exceeds the activation energy.

(c) the pre-exponential factor is a measure of the rate at which collisions occur, irrespective of their energy.

(d) higher the magnitude of activation energy, stronger is the temperature dependence of the rate constant

Explanation:

a. High activation energy usually implies a **slow reaction**.

b. Rate constant of a reaction increases with increase in temperature due to increase in number of collisions whose energy exceeds the activation energy.

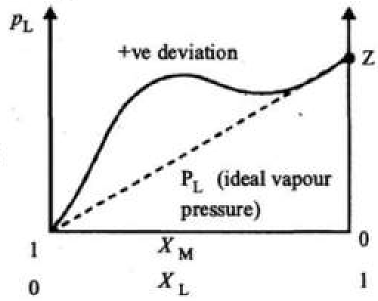
c. $k = P \times Z \times e^{-Ea/RT}$

d. So, pre-exponential factor (A) = P × Z and it is independent of activation energy or energy of molecules,

22. **(a)** The point Z represents vapour pressure of pure liquid L and Raoult's law is obeyed when $x_L \rightarrow 1$.

(c) Attractive intermolecular interactions between L-L in pure liquid L and M-M in pure liquid M are stronger than those between L-M when mixed in solution.

Explanation:



i. From the given figure it is clear that at point Z, the mole fraction of M is zero, in that case 'Z' represents the vapour pressure of pure liquid L and Raoulf s law is obeyed when $x_L \rightarrow 1$.

ii. The solution formed by mixing two liquids L and M shows positive deviation from Raoult's law. Thus, intermolecular forces of attraction between L-L in pure liquid L and M-M in pure liquid M are stronger than that between L-M in the solution.

23. **(a)** Na_2O

(b) O_2

(d) N_2

Explanation: Sodium nitrate on decomposition upto $500^{o}C$ gives $NaNO_2$ and oxygen.

$2NaNO_3 \xrightarrow{\Delta} 2NaNO_2 + O_2 \uparrow$

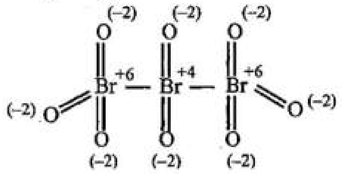
While at higher temperature (i.e. above $800^{o}C$), $NaNO_2$ further decomposes into Na_2O, N_2 and O_2.

$2NaNO_2 \xrightarrow{800^{\circ}C} Na_2O + \frac{3}{2}O_2 \uparrow + N_2 \uparrow$

24. 6

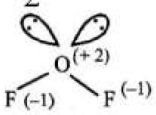
Explanation:

Br_3, O_8



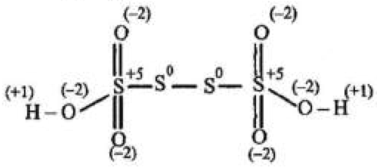
Number of atoms with zero oxidation state = 0

F_2O



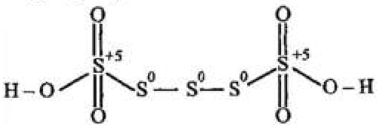
Number of atoms with zero oxidation state = 0

$H2S_4O_6$



Number of atoms with zero oxidation state = 2

$H_2S_5O_6$



Number of atoms where zero oxidation state = 3

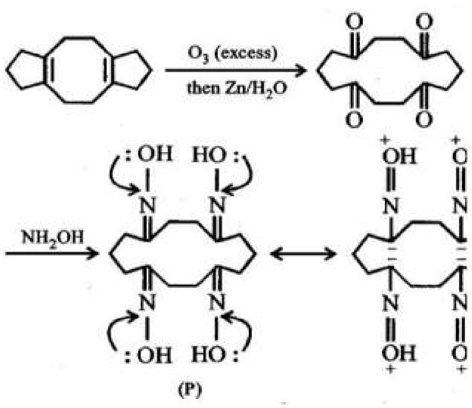
C_3O_2

Number of atoms with zero oxidation state = 1

Sum = 2 + 3 + 1 = 6

25. 12.0

Explanation:



So, total 12 atoms are sp^2 hybridised (4 C atoms, 4 N atoms and 4 O atoms)

26. 24.0

Explanation:

$\overset{(+6)}{Cr}_2O_7^{2-} \rightarrow Cr^{3+}$

$Fe^{2+} \rightarrow Fe^{3+}$

Equivalents of $K_2Cr_2O_7$ = Eq. of Fe^{2+}

(Molarity $\times$ n.f. $\times$ volume) of $K_2Cr_2O_7 \equiv$ (Molarity $\times$ n.f. $\times$ volume) of Fe^{2+}

$\Rightarrow 0.02 \times 6 \times 20 = M \times 1 \times 10 \Rightarrow M = 0.24 = 24 \times 10^{-2}$

27. 117.0

Explanation:

117

28. 1.344

Explanation:

Meq. of H_2O_2 = Meq. of $Na_2S_2O_3$ = Meq. of I_2 = Meq.of KI

$\frac{w}{17} \times 1000 = 20 \times 0.3$

$\therefore$ w = 0.102 g (equating Meq. in 25 mL solution)

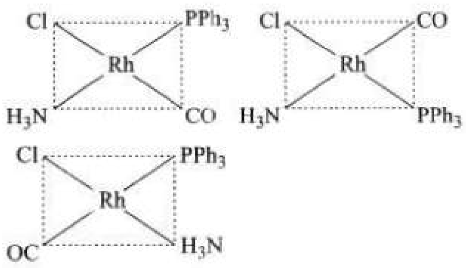
$H_2O_2 \longrightarrow H_2O + \frac{1}{2}O_2$

$\therefore$ Volume of $O_2 = \frac{11200 \times 0.102}{34} = 33.6$ mL

$\therefore$ Volume strength $= \frac{33.6}{25} = 1.344$

29. 3

Explanation:



30.

(d) P - 1; Q - 5; R - 4; S - 1

Explanation:

P - 1; Q - 5; R - 4; S - 1

31.

(d) A - III, B - I, C - IV, D - II

Explanation:

A - III, B - I, C - IV, D - II

32.

(c) (P) - (3), (Q) - (4), (R) - (2), (S) - (1)

Explanation:

(P) $\underset{X}{(C_2H_5)_3N} + \underset{Y}{CH_3COOH} \longrightarrow (C_2H_5)_3NH^+CH_3COO^-$

Initially conductivity increases because on neutralisation ions are created. After that it becomes practically constant because X alone cannot form ions.

(Q) $\underset{X}{KI}(0.1M) + \underset{Y}{AgNO_3(0.01M)} \rightarrow AgI \downarrow + KNO_3$

Number of ions in the solution remains constant as only $AgNO_3$ precipitated as AgI. Thereafter, conductance increases due to increase in number of ions.

(R) Initially conductance decreases due to the decrease in the number of $\bar{O}H$ ions as OH^- is getting replaced by CH_3COO^- which has poorer conductivity. Thereafter, it slowly increases due to the increase in number of H^+ ions.

(S) Initially it decreases due to decrease in H^+ ions and then increases due to the increase in OH^- ions.

Maths

33.

(d) $(-3, \infty) - \{-1, -2\}$

Explanation:

Given, $f(x) = \frac{\log_2(x+3)}{(x^2+3x+2)} = \frac{\log_2(x+3)}{(x+1)(x+2)}$

For numerator, $x + 3 > 0$

$\Rightarrow$ x > -3 ..... (i)

and for denominator, (x + 1) (x + 2) $\neq$ 0

$\Rightarrow x \neq -1, -2$ .... (ii)

From eqs. (i) and (ii)

Domain is $(-3, \infty)/\{-1, -2\}$

34.

(c) directrix 4x = 3

Explanation:

Given, $y^2 = 24x \Rightarrow a = 6$ and xy = 2

AB $\equiv$ $ty = x + 6t^2$ ... (i)

AB $\equiv$ T = S_1

kx + hy = 2hk ...(ii)

From (i) and (ii)

$\frac{k}{1} = \frac{h}{-t} = \frac{2hk}{-6t^2} \Rightarrow k = \frac{h}{-t}, t = \frac{k}{3} \Rightarrow k^2 = -3h$

Then locus is $y^2 = -3x$

Therefore, directrix is 4x = 3

35.

(d) f(x) is strictly increasing function

Explanation:

Given, $f(x) = x^3 + bx^2 + cx + d$

$\Rightarrow f'(x) = 3x^2 + 2bx + c$

As we know that, if $ax^2 + bx + c > 0, \forall x$, then a > 0 and D < 0.

Now, $D = 4b^2 - 12c = 4(b^2 - c) - 8c$ [where, $b^2 - c < 0$ and c > 0]

$\therefore$ D = (-ve) or D < 0

$\Rightarrow f'(x) = 3x^2 + 2bx + c > 0 \forall x \in (-\infty, \infty)$ [as D < 0 and a > 0]

Hence, f(x) is strictly increasing function.

36.

(c) 7

Explanation:

We know that,

$(a+b)^5 + (a-b)^5 = {}^5C_0a^5 + {}^5C_1a^4b + {}^5C_2a^3b^2$

$+ {}^5C_3a^2b^3 + {}^5C_4ab^4 + {}^5C_5b^5 + {}^5C_0a^5 - {}^5C_1a^4b + {}^5C_2a^3b^2 - {}^5C_3a^2b^3 + {}^5C_4ab^4 - {}^5C_5b^5$

$2[a^5 + 10a^3b^2 + 5ab^4]$

$\therefore [x + x^3 - 1)^{1/2}]^5 + [x - (x^3 - 1)^{1/2}]^5$

$= 2\left[x^5 + 10x^3\left(x^3 - 1\right) + 5x\left(x^3 - 1\right)^2\right]$

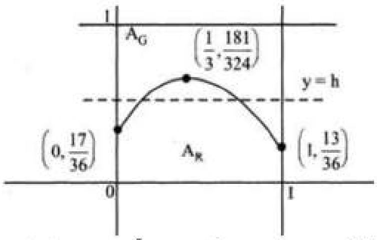
Therefore, the given expression is a polynomial of degree 7.

37. **(a)** Statement (b) is true.

(b) Statement (c) is true.

(c) Statement (d) is true.

Explanation: Given function,



$f(x) = \frac{x^3}{3} - x^2 + \frac{5x}{9} + \frac{17}{36}$

$f'(x) = x^2 - 2x + \frac{5}{9} = 0 \Rightarrow x = \frac{1}{3}$

$f''(x) = 2x - 2 = \frac{2}{3} - 2 < 0$ at $x = \frac{1}{3}$

$\therefore$ f(x) is maximum at $x = \frac{1}{3}$

Now, $A_R = \int_0^1 f(x)dx = \int_0^1 \left(\frac{x^3}{3} - x^2 + \frac{5}{5}x + \frac{17}{36}\right) dx = \frac{1}{2}$

$\therefore A_G = 1 - \frac{1}{2} = \frac{1}{2}$

a. Since the area of the green region above the line L_h equals the area of the green region below the line L_h.

$\Rightarrow 1 - h = h - \frac{1}{2} \Rightarrow h = \frac{3}{4}, \frac{3}{4} > \frac{2}{3}$

So statement (a) is incorrect

b. Since the area of the red region above the line L_h equals the area of the red region below the line L_h.

$\Rightarrow h = \frac{1}{2} - h \Rightarrow h = \frac{1}{4}$

so statement (b) is correct

c. Since the area of the green region above the line L_h equals the area of the red region below the line L_h.

When $h = \frac{181}{324}, A_R = \frac{1}{2}, A_G < \frac{1}{2}$

$h = \frac{13}{36}, A_R = \frac{1}{2}, A_G < \frac{1}{2}$

$A_R = A_G$ for some $\left(\frac{13}{36}, \frac{181}{324}\right)$

so statement (c) is correct

d. $\because$ statement (c) is correct $\Rightarrow$ statement (d) is also correct

38. **(a)** $2\alpha - \beta + 2\gamma + 4 = 0$

(b) $2\alpha - \beta + 2\gamma - 8 = 0$

Explanation: $P_3 :(x + z - 1) + \lambda y = 0 \Rightarrow x + \lambda y + z - 1 = 0$

Distance of point (0, 1, 0) from P_3 :

$\left|\frac{\lambda-1}{\sqrt{2+\lambda^2}}\right| = 1 \Rightarrow \lambda^2 - 2\lambda + 1 = \lambda^2 + 2 \Rightarrow \lambda = \frac{-1}{2}$

Distance of point (α, β, γ) from P_3 :

$\left|\frac{\alpha+\lambda\beta+\gamma-1}{\sqrt{2+\lambda^2}}\right| = 2 \Rightarrow \frac{\alpha-\frac{1}{2}\beta+\gamma-1}{\frac{3}{2}} = \pm 2$

$\Rightarrow \alpha - \frac{1}{2}\beta + \gamma - 1 = \pm 3 \Rightarrow 2\alpha - \beta + 2\gamma - 2 = \pm 6$

$\Rightarrow 2\alpha - \beta + 2\gamma - 8 = 0$ or $2\alpha - \beta + 2\gamma + 4 = 0$

39. **(a)** $n_2 = 44$

(b) $n_1 = 1000$

(d) $\frac{n_4}{12} = 420$

Explanation: Number of elements is $S_1 = 10 \times 10 \times 10 = 1000$

Number of elements is $S_2 = 9(J = 8) + 8(J = 7) + 7(J = 6) + 6(J = 5) + 5(J = 4) + 4(J = 3) + 3(J = 2) + 2(J = 1)$

$= 44$

Number of elements in $S_3 = {}^{10}C_4 = 210$

Number of elements in $S_4 = {}^{10}P_4 = 210 \times 4! = 5040$

So, options $(n_1 = 1000)$, $(n_2 = 44)$, $(\frac{n_4}{12} = 420)$ are correct.

40. 7

Explanation:

$\lim_{x\to 1} \frac{F(x)}{G(x)} = \frac{1}{14} \Rightarrow \lim_{x\to 1} \frac{\int_{-1}^{x} f(t)dt}{\int_{-1}^{x} t|f(f(t))|dt}$

$\because \int_{-1}^{1} f(t)dt = 0$ and $\int_{-1}^{1} t|f(f(t))|dt = 0$

f(t) being odd function

$\therefore$ Using L Hospital's rule, we get

$\lim_{x\to 1} \frac{f(x)}{x|f(f(x))|} = \frac{1}{14}$

$\Rightarrow \frac{f(1)}{|f(f(1))|} = \frac{1}{14} \Rightarrow \frac{\frac{1}{2}}{\left|f\left(\frac{1}{2}\right)\right|} = \frac{1}{14}$

$\Rightarrow \left|f\left(\frac{1}{2}\right)\right| = 7 \Rightarrow f\left(\frac{1}{2}\right) = 7$

41. 42

Explanation:

Let the equation of line be given by $y = mx + c$

Take the probability distribution table.

x	0	1	2	3	4
$P(x)$	c	$m+c$	$2m+c$	$3m+c$	$4m+c$

$P(X=x)=0 \forall x \in R-\{0,1,2,3,4\}$

Now, $\sum_{x=0}^{4}(mx+c)=1 \Rightarrow 10m+5c=1$ ...(i)

Mean = $=\sum x_i p_i=\sum_{x=0}^{4}(mx+c)x$

$=30m+10c=\frac{5}{2}$ (Given) ...(ii)

Using (i) and (ii), $m=\frac{1}{20}, c=\frac{1}{10}$

$\sum p_i x_i^2=\sum_{x=0}^{4}(mx+c)x^2=$ 100m + 30c = 5+3 = 8

Variance $=\sum p_i x_i^2-\left(\sum p_i x_i\right)^2=8-\left(\frac{5}{2}\right)^2=\frac{7}{4}$

Thus. $24\alpha=42$

42. 1.0

Explanation:

$z=\frac{-1+i\sqrt{3}}{2} \Rightarrow z^3=1$ and $1+z+z^2=0$

$P^2=\begin{bmatrix}(-z)^r & z^{2s}\\ z^{2s} & z^r\end{bmatrix}\begin{bmatrix}(-z)^r & z^{2s}\\ z^{2s} & z^r\end{bmatrix}$

$=\begin{bmatrix}z^{2r}+z^{4s} & z^{2s}\left((-z)^r+z^r\right)\\ z^{2s}\left((-z)^r+z^r\right) & z^{4s}+z^{2r}\end{bmatrix}$

For P^2 = -I, we should have

$z^{2r}+z^{4s}=-1$ and $z^{2s}((-z)^r+z^r)=0$

$\Rightarrow z^{2r}+z^{4s}+1=0$ and $(-z)^r+z^r=0$

$\Rightarrow$ r is odd and s = r but not a multiple of 3,

which is possible when s = r = 1

$\therefore$ only one pair is there.

43. 6

Explanation:

Let the sides are a - d, a and a + d. Then,

a(a - d) = 48

and $a^2-2ad+d^2+a^2=a^2+2ad+d^2$

$\Rightarrow a^2=4ad$

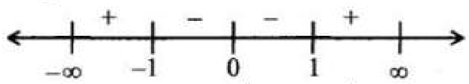
$\Rightarrow$ a = 4d

Thus, a = 8, d = 2

Hence a - d = 6

44. 4.0

Explanation:



$3x^2 + x - 1 = 4\left|x^2 - 1\right|$

Case 1: If $x \in [-1, 1]$

$3x^2 + x - 1 = -4x^2 + 4$

$\Rightarrow 7x^2 + x - 5 = 0 \because D = 141 > 0$

$\therefore$ Equation has two roots

Case 2: If $x \in (-\infty, -1] \cup [1, \infty)$

$3x^2 + x - 1 = 4x^2 - 4$

$\Rightarrow x^2 - x - 3 = 0 \because D = 13 > 0$

$\therefore$ Equation has two roots So, total 4 roots

45. 20

Explanation:

$f(x) = x^4 + ax^3 + bx^2 + c$

$f(1) = -9 \Rightarrow a + b + c = -10$

$4x^3 + 3ax^2 + 2bx = 0$ has roots $\sqrt{3}i, -\sqrt{3}i, 0$

So, $a = 0$ and $\frac{2b}{4} = (\sqrt{3}i)(-\sqrt{3}i) = 3.$

$\therefore b = 6.$ So, $c = -16$

$f(x) = x^4 + 6x^2 - 16 = \left(x^2 + 8\right)\left(x^2 - 2\right)$

Roots are $\pm\sqrt{8}i, \pm\sqrt{2}$

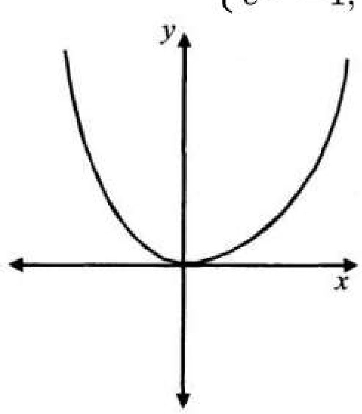
$|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 = 2 \cdot 8 + 2 \cdot 2 = 20$

46.

(c) P $\rightarrow$ 1; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 4

Explanation:

$P(1) : f_4(x) = \begin{cases} x^2, & x < 0 \\ e^{2x} - 1, & x \geq 0 \end{cases}$

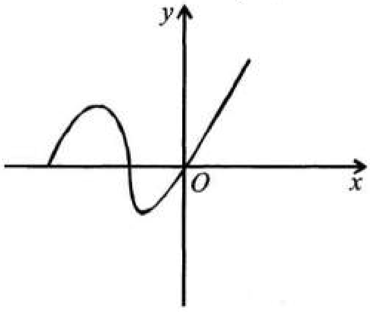


Range of $f_4 = [0, \infty)$

$\therefore$ f_4 is onto.

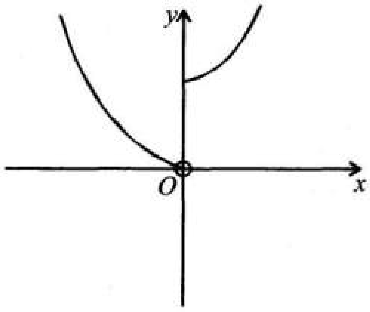
From graph f_4 is not one one.

$$Q(3) : f_3(x) = \begin{cases} \sin x, & x < 0 \\ x, & x \geq 0 \end{cases}$$



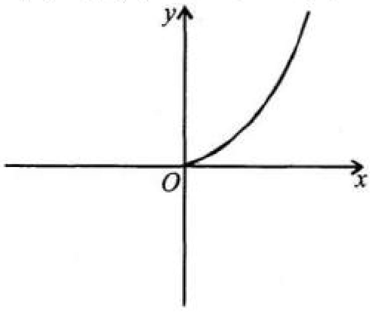
From graph f is differentiable but not one one.

$$R(2) : f_2 0 f_1(x) = \begin{cases} x^2, & x < 0 \\ e^{2x}, & x \geq 0 \end{cases}$$



From graph $f_2 0 f_1$ is neither continuous nor one one.

S(4): $f_2(x) = x^2, x \in [0, \infty)$



It is continuous and one one.

47. **(a)** (P) $\to$ (3), (Q) $\to$ (2), (R) $\to$ (4), (S) $\to$ (5)

Explanation:

Given

x_i	3	4	5	8	10	11
f_i	5	4	4	2	2	3

x_i	f_i	$x_i f_i$	C.F.	x_i - Mean	$f_i\|x_i$ - Mean$\|$	$\|x_i$ - Median$\|$	$f_i\|x_i$ - Median$\|$
3	5	15	5	3	15	2	10
4	4	16	9	2	8	1	4
5	4	20	13	1	4	0	0
8	2	16	15	2	4	3	6
10	2	20	17	4	8	5	10
11	3	33	20	5	15	6	18
	$\Sigma f_i =$ 20	$\Sigma x_i f_i =$ 120			$\Sigma f_i \mid x_i -$ Mean $\mid =$ 54		$\Sigma f_i \mid x_i -$ Median $\mid =$ 48

P. Mean $= \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{120}{20} = 6$

Q. Median $= \left(\frac{N}{2}\right)^{th}$ obs. $\left(\frac{20}{2}\right)^{th}$ obs. 10^{th} obs. = 5

R. Mean deviation about mean

$= \frac{\Sigma f_i |x_i - \text{Mean}|}{\Sigma f_i} = \frac{54}{20} = 2.70$

S. Mean deviation about median

$= \frac{\Sigma f_i |x_i - \text{Median}|}{\Sigma f_i} = \frac{48}{20} = 2.40$

48.

(b) (P) $\rightarrow$ (2), (Q) $\rightarrow$ (1), (R) $\rightarrow$ (3), (S) $\rightarrow$ (5)

Explanation:

Given, $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$ ...(i)

$|\bar{z}|^3 + 2\bar{z}^2 + 4z - 8 = 0$ [Conjugate both sides]

$2(z^2 - \bar{z}^2) + 4(\bar{z} - z) = 0$

$\Rightarrow 2(z - \bar{z})[z + \bar{z} - 2] = 0$

$\because z = \bar{z}$ (Not possible) or $z + \bar{z} = 2$

$\therefore z = 1 + bi(b \neq 0) \Rightarrow \bar{z} = 1 - bi$

$(1 + b^2)^{\frac{3}{2}} + 2(1 - b^2 + 2bi) + 4(1 - bi) - 8 = 0$ [from (i)]

$(1 + b^2)^{\frac{3}{2}} - 2(1 + b^2) = 0$

$\Rightarrow (1 + b^2)\left(\sqrt{1 + b^2} - 2\right) = 0$

$\because 1 + b^2 \neq 0 \Rightarrow \sqrt{1 + b^2} - 2 = 0 \Rightarrow b^2 = 3$

P. $|z|^2 = 1 + b^2 = 1 + 3 = 4$

Q. $|z - z|^2 = |1 + ib - 1 + ib|^2 = 4b^2 = 12$

R. $|z|^2 + |z + \bar{z}|^2 = 4 + |1 + ib + 1 - ib|^2 = 4 + 4 = 8$

S. $|z + 1|^2 = |1 + 1 + ib|^2 = 4 + b^2 = 4 + 3 = 7$